$$+\frac{5a^3}{128}\sin^3\theta\cos^3\theta+\frac{63a^5}{8192}\sin^5\theta\cos^5\theta+\ldots\Big]\frac{\cos\theta\,d\theta}{\sqrt{(\sin\theta)}}.$$

$$\frac{1}{\sqrt{(\sin\theta)}}=(1-\cos^2\theta)^{-\frac{1}{4}}.$$

$$\therefore I=1+\frac{31}{15}\cdot\frac{a^2}{2^9}+\frac{85015}{1989}\cdot\frac{a^4}{2^{19}}+\frac{2350494}{38675}\cdot\frac{a^6}{2^{28}}+\ldots$$

$$+\frac{1}{\sqrt{2}}\left[\frac{a}{35.\,2^5}\left(\frac{341}{3}-\frac{20611\sqrt{3}}{2^9}+\ldots.\right)+\frac{a^3}{21.\,2^8}\left(\frac{41}{3}-\frac{2616\sqrt{3}}{2^{10}}+\ldots.\right)\right.$$

$$\left.+\frac{a^5}{11.\,2^{10}}\left(1-\frac{8181\sqrt{3}}{2^{14}}+\ldots.\right)\right]+\ldots.$$

a cannot be greater than $\frac{8}{3}\sqrt{3}$.

This solution, to be complete, should have investigated the matter of convergency and, since the function vanishes at the lower limit, also the condition of determinateness.

The proposer of 259 (256) suggests that the equation be changed to $(1+y+2axy)dx+x(1+x)dy=0$.

MECHANICS.

211. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, England.

A smooth elliptic wire, axis vertical, has a small ring sliding on it, connected by elastic strings with each focus. Either string is just unstretched when the ring is nearest the corresponding focus. The modulus of elasticity is W/n, where W oz. is the weight of the ring. Find the distance of the ring from the upper focus in the different positions of equilibrium, and in each case discuss the nature of the equilibrium.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let F be the upper focus of the elliptic wire, semi-major axis a, eccentricity e. Let r, $2r-a$ be the lengths of the strings from the upper and lower foci to the ring at the point P on the curve; G the intersection of the normal from P with the axis major; θ the angles the strings make with the tangent at P; ϕ the angle the tangent at P makes with the major axis; T the tension of string r; Q the tension of string $2a-r$; m the weight of the ring and strings that cause a downward force due to gravity; $a(1-e)$=unstretched length of each string.

Then $PF=r$, $GF=er$, $\sin GPF=\cos\theta$, $\sin PGF=\cos\phi$, $\sin PGF:\sin GPF=r:er$. $\therefore\cos\phi/\cos\theta=1/e$.

Also $r=a(1-e)(1+Tn/W)$, $2a-r=a(1-e)(1+Qn/W)$.

$$\therefore T=\frac{W(r-a+ae)}{an(1-e)},\quad Q=\frac{W(a+ae-r)}{an(1-e)}.$$

I. For equilibrium, $T\cos\theta=Q\cos\theta+m\cos\phi$.

$$\therefore \frac{T-Q}{m}=\frac{\cos\phi}{\cos\theta}=\frac{1}{e}.\quad \therefore \frac{2W(r-a)}{amn(1-e)}=\frac{1}{e}.$$

$$\therefore r=a+\frac{amn(1-e)}{2eW}=a+\frac{an\,(1-e)}{2e},\text{ when } m=W.$$

When the ring is displaced it will tend to regain this same position of equilibrium.

II. For equilibrium, $T\cos\theta=Q\cos\theta=0$.

$\therefore r=a(1-e)$ and $r=a(1+e)$, or the upper and lower vertices. When the ring is displaced from either of these positions it will tend to equilibrium in I.

212. Proposed by W. J. GREENSTREET, M. A., Marling School, Stroud, Eng.

A peg A is vertically d feet above a peg B. A string AD, a feet long, with two equal, jointed rods DC, CB form the whole figure. Discuss the position of equilibrium.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let θ=the angle the string makes with the vertical; ϕ=the angle DC makes with the vertical; ψ=the angle CB makes with the vertical; b=the length of each rod; W its weight. Also regard the string as weightless, and let x=the depth of the center of gravity of the system below A.

$$\therefore x=[w(a\cos\theta+\tfrac{1}{2}b\cos\phi)+w(d\pm\tfrac{1}{2}b\cos\psi)]/2w\ldots(1).$$

Projecting vertically, we get $a\cos\theta+b\cos\phi\mp b\cos\psi=d\ldots(2)$.

$$\text{Also, } a^2+d^2-2ad\cos\theta=2b^2-2b^2\cos(\psi-\phi)\ldots(3).$$

$a\cos\theta$ from (2) in (1) and (3) gives $x=[w(2d-\frac{1}{2}b\cos\phi\pm\frac{3}{2}b\cos\psi)]/2w\ldots(4)$.

$$a^2-d^2+2db\cos\phi\mp2db\cos\psi=2b^2-2b^2\cos(\psi-\phi)\ldots(5).$$

Differentiating (4) and (5), we get $3\sin\psi\,d\psi=\pm\sin\phi\,d\phi\ldots(6)$.

$$[b\sin(\psi-\phi)\pm d\sin\psi]d\psi=[b\sin(\psi-\phi)-d\sin\phi]d\phi\ldots(7).$$

Eliminating dx and $d\phi$ between (6) and (7),

$$b\sin(\psi-\phi)(3\sin\psi\pm\sin\phi)=2d\sin\phi\sin\psi\ldots(8).$$

(5) and (8) determine the equilibrium. The $\pm$ sign is used as follows: if a is long enough to permit C to fall below B use the upper sign; if not, use the lower.